Demo AS Discrete Questions

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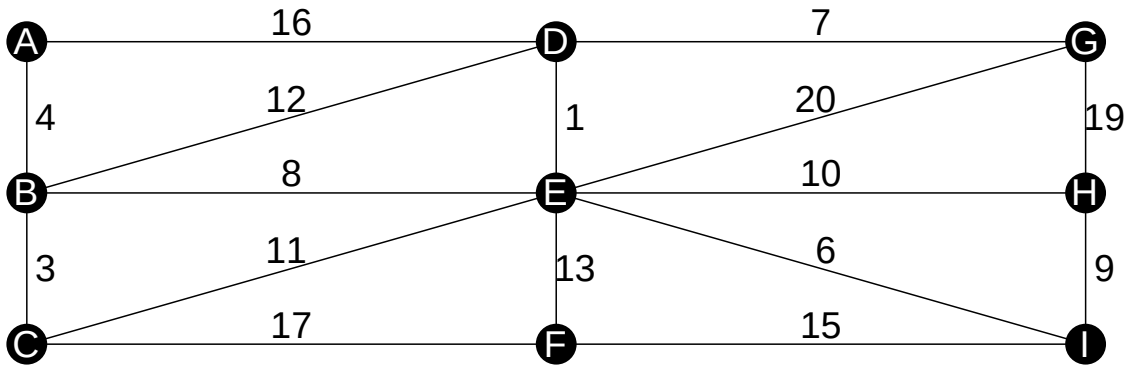
Please answer on file paper.

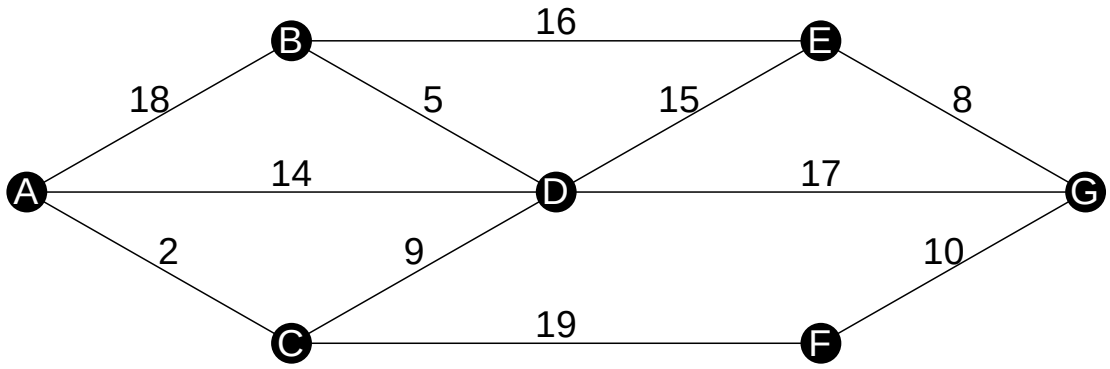
1: Find the Minimum Spanning Tree using Prim's Algorithm starting from vertex A:

a)



Arcs/Length:

b)



Arcs/Length:

2: Find the Minimum Spanning Tree using Prim's Algorithm starting from vertex A:

a)

	A	B	C	D	E	F	G
A	–	19	12	17	23	21	30
B	19	–	7	11	20	10	4
C	12	7	–	1	8	9	3
D	17	11	1	–	13	6	25
E	23	20	8	13	–	26	22
F	21	10	9	6	26	–	18
G	30	4	3	25	22	18	–

Arcs:

Total length=

b)

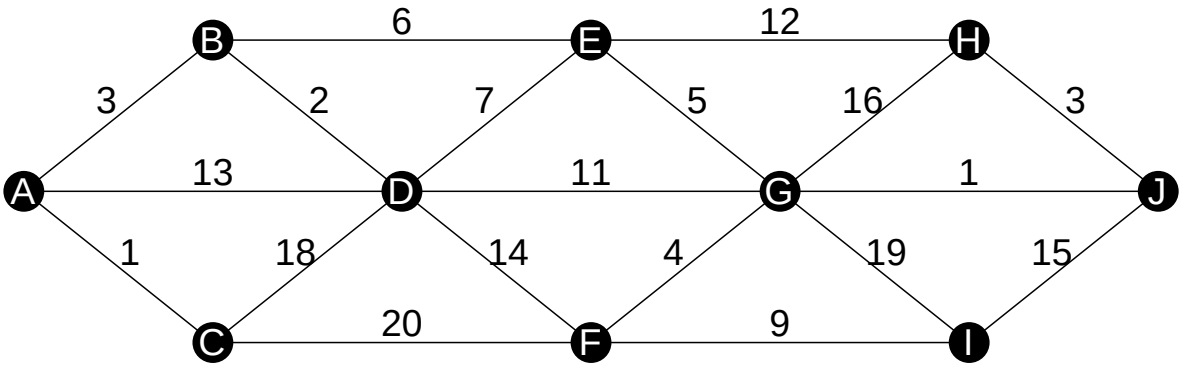
	A	B	C	D	E	F
A	–	2	27	14	24	16
B	2	–	15	5	29	28
C	27	15	–	11	3	12
D	14	5	11	–	4	21
E	24	29	3	4	–	9
F	16	28	12	21	9	–

Arcs:

Total length=

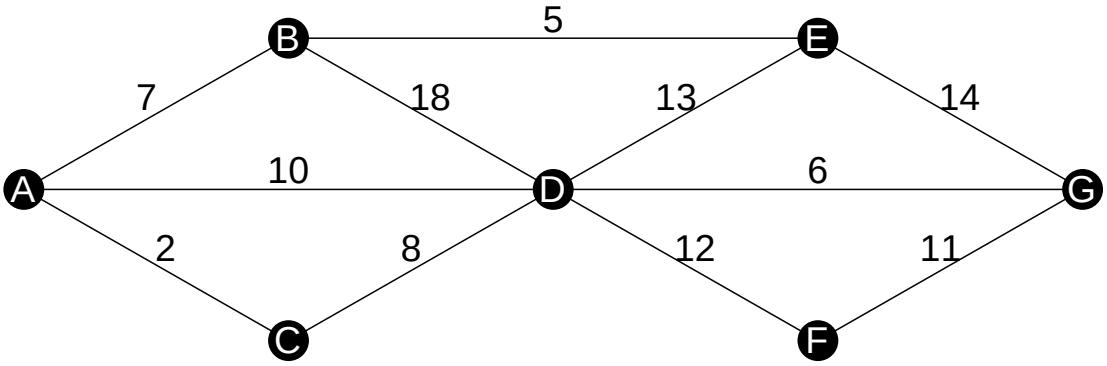
3: Find the Minimum Spanning Tree using Kruskal's Algorithm:

a)



Arcs/Length:

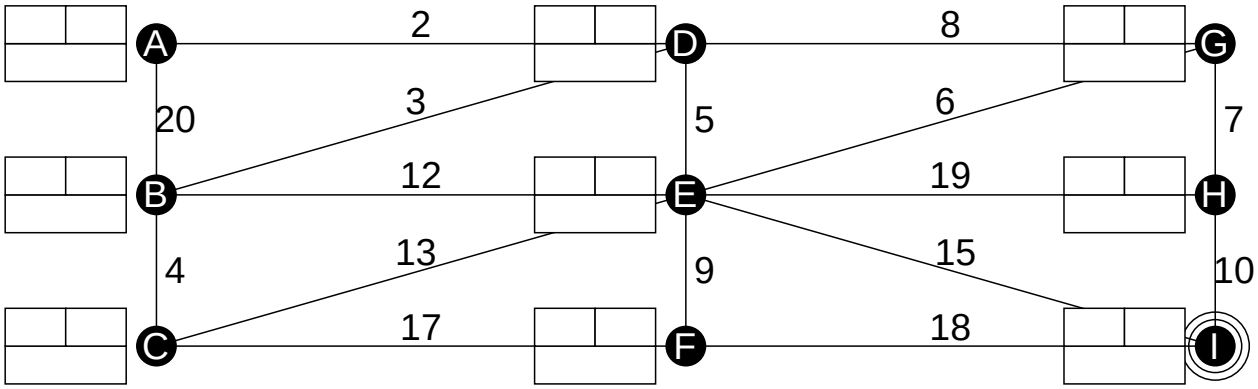
b)



Arcs/Length:

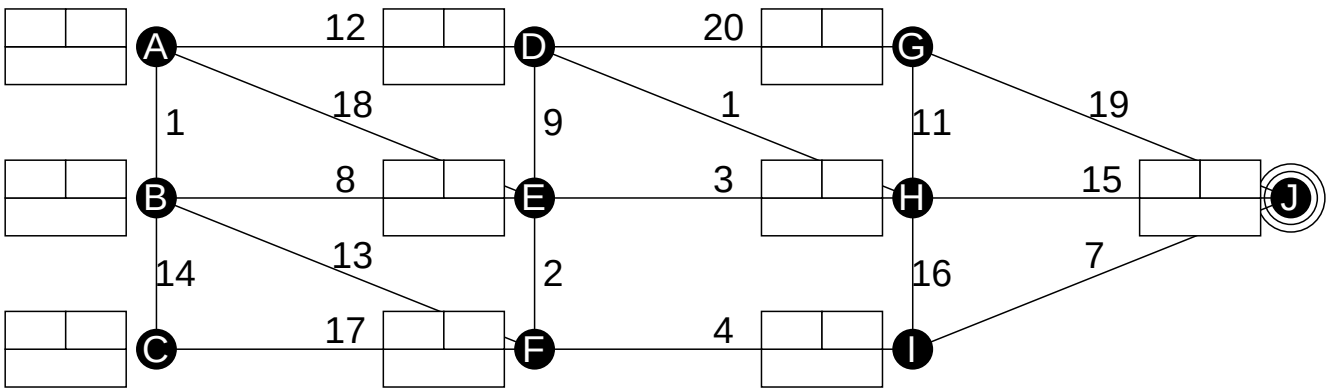
4: Find the shortest route from A to the ringed vertex using Dijkstra's Algorithm:

a)



Route/Length:

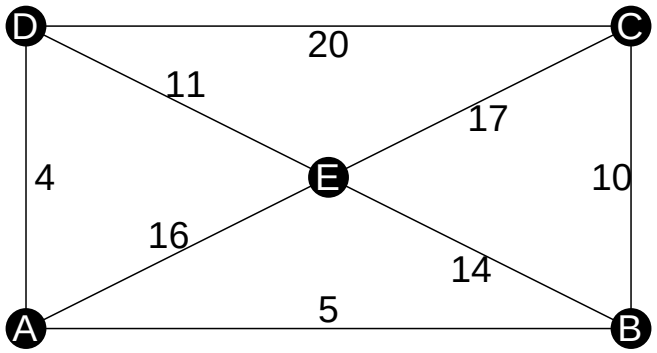
b)



Route/Length:

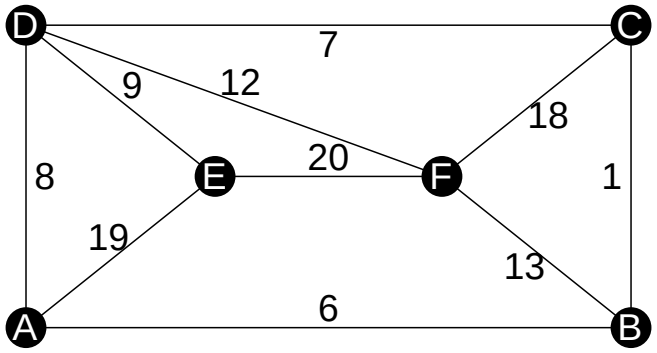
5: Find the shortest route starting and finishing at A which includes every edge.

a)



Pairings/Best solution:

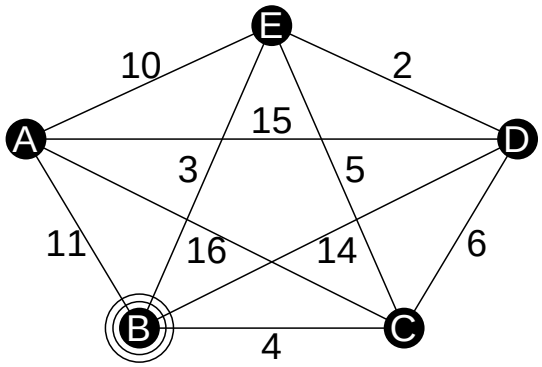
b)



Pairings/Best solution:

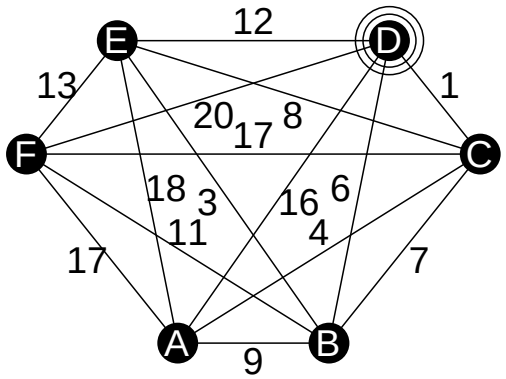
6: Find a Hamiltonian cycle using the Nearest Neighbour algorithm (start at the ringed vertex):

a)



Cycle/Length:

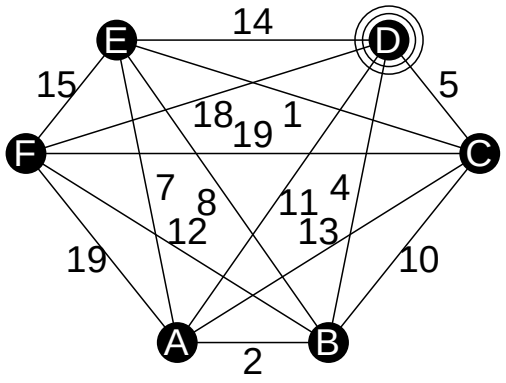
b)



Cycle/Length:

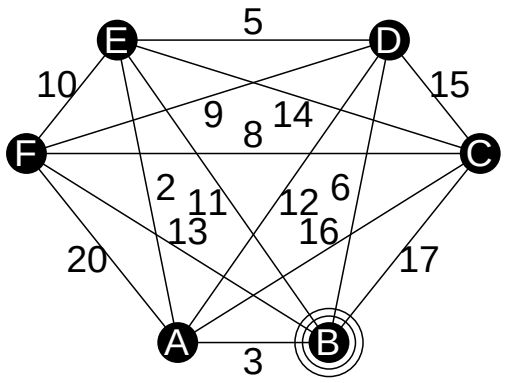
7: Find a lower bound for the Travelling Salesperson Problem (remove the ringed vertex):

a)



Lower Bound:

b)



Lower Bound:

8: Use the first fit algorithm to pack these objects into bins:

a) 6 6 2 3 12 11 1 19
(Bin size 20)

b) 2 1 1 9 6 7 4 6
(Bin size 10)

9: Use the first fit decreasing algorithm to pack these objects into bins:

a) 8 14 14 4 4 3 8 2 4 2 13 8 6
(Bin size 18)

b) 6 2 3 4 3 3 6 4 8 9
(Bin size 12)

10: Use the full bin algorithm to pack these objects into bins:

a) 5 1 9 6 8 1 3
(Bin size 11)

b) 5 6 4 3 2 1 1 8 9 6 13 8 7
(Bin size 14)

11: Arrange the following lists in ascending order using Bubble Sort:

a) 3 6 2 7 13 10

b) 14 5 15 8 10 2 11

12: Arrange the following lists in ascending order using Shuttle Sort:

a) 7 3 5 13 10

b) 3 11 9 14 2 6

13: Find the order of each algorithm, given its efficiency:

a) Time taken = $6^n + 19n^2 + n^4 + 20n^5 + 6n!$

b) Time taken = $10n^9$

14: Solve the following:

a) An algorithm has order n and takes 5 seconds to solve a problem of size 1.
Estimate the time taken to solve a problem of size 8.

b) An algorithm has order 3^n and takes 648 seconds to solve a problem of size 4.
Estimate the time taken to solve a problem of size 10.

15: Give the following Linear Programming terms:

a) Find the smallest value of

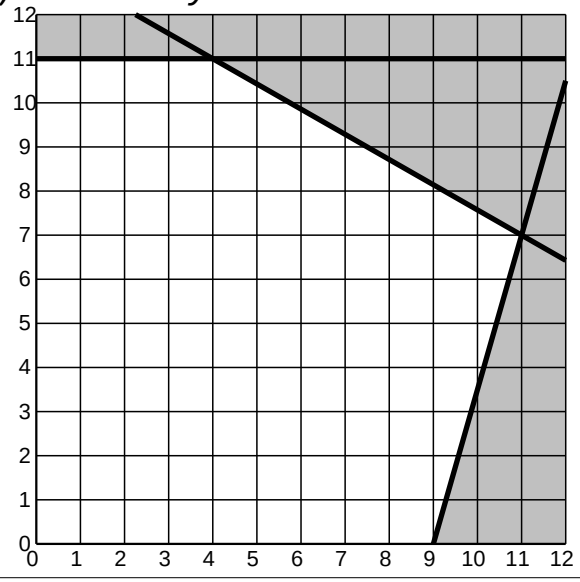
b) The statement of a problem as a function to be optimised, with a set of inequalities

c) The function to be optimised (minimised or maximised)

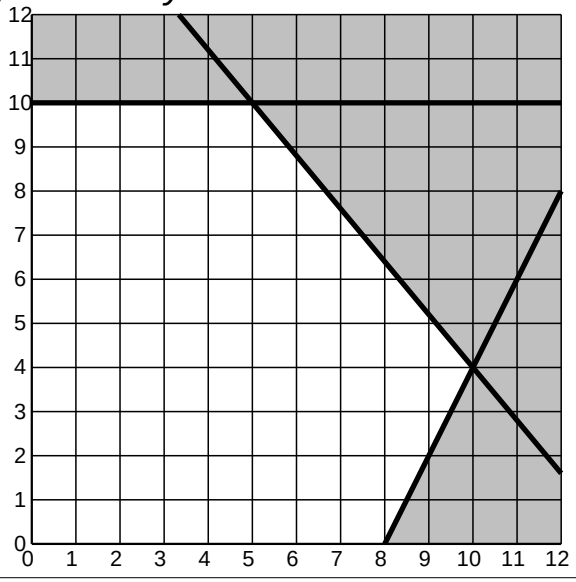
d) A quantity that can be changed in the problem (e.g. representing a physical quantity)

16: Maximise each objective function:

a) $P = 2x + 5y$



b) $P = 2x + y$



17: Solve using the Simplex algorithm:

a)

P	x	y	s	t	value
0	–4	3	1	0	6
0	6	3	0	1	36
1	–36	–18	0	0	32

b)

P	x	y	s	t	value
0	2	8	1	0	64
0	6	–7	0	1	6
1	–18	–40	0	0	27

18: Solve using the Simplex algorithm:

a)

P	x	y	z	s	t	u	value
0	4	30	15	1	0	0	6
0	4	20	12	0	1	0	27
0	8	4	36	0	0	1	36
1	–6	–14	–9	0	0	0	36

b)

P	x	y	z	s	t	u	value
0	10	24	10	1	0	0	40
0	16	24	40	0	1	0	2
0	9	10	9	0	0	1	21
1	–5	–15	–15	0	0	0	10

Answers: Demo AS Discrete Questions

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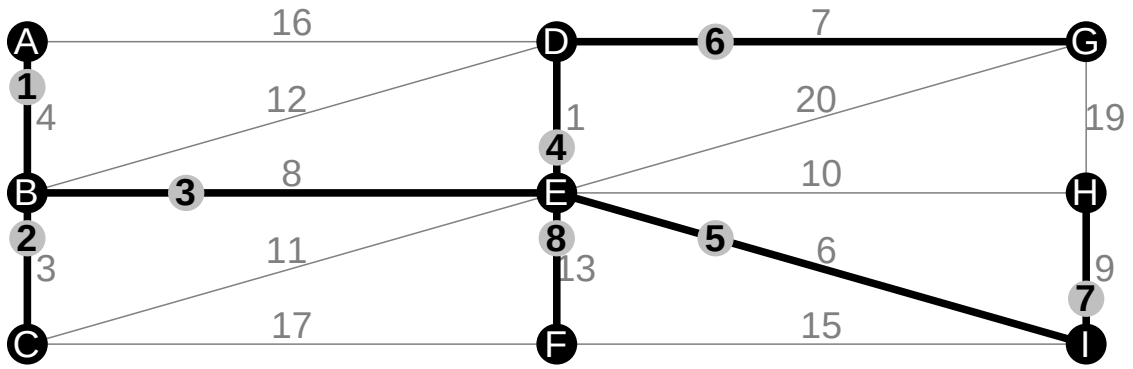
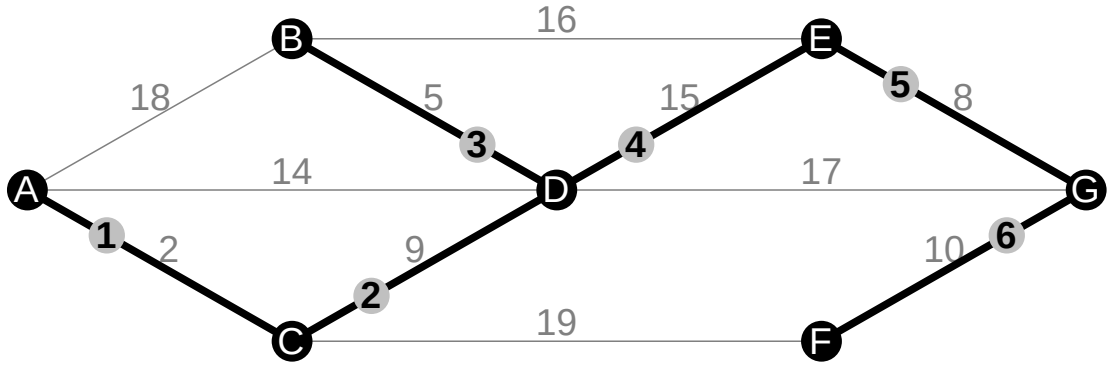
1: a)



Arcs: AB, BC, BE, ED, EI, DG, IH, EF. Total length=51

b)



Arcs: AC, CD, DB, DE, EG, GF. Total length=49

2: a)

	A (1)	B (5)	C (2)	D (3)	E (7)	F (6)	G (4)
A	–	19	12	17	23	21	30
B	19	–	7	11	20	10	(4)
C	(12)	7	–	1	8	9	3
D	17	11	(1)	–	13	6	25
E	23	20	(8)	13	–	26	22
F	21	10	9	(6)	26	–	18
G	30	4	(3)	25	22	18	–

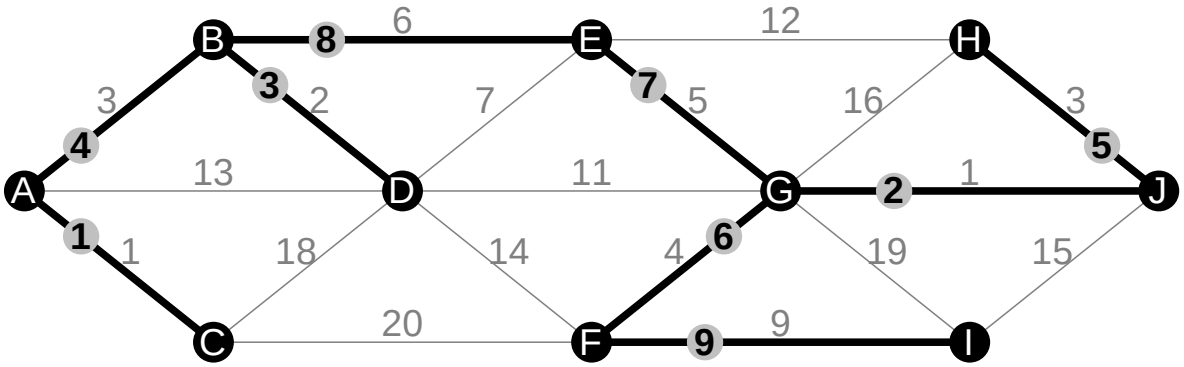
Arcs: AC, CD, CG, GB, DF, CE.
Total length=34

b)

	A (1)	B (2)	C (5)	D (3)	E (4)	F (6)
A	–	2	27	14	24	16
B	(2)	–	15	5	29	28
C	27	15	–	11	(3)	12
D	14	(5)	11	–	4	21
E	24	29	3	(4)	–	9
F	16	28	12	21	(9)	–

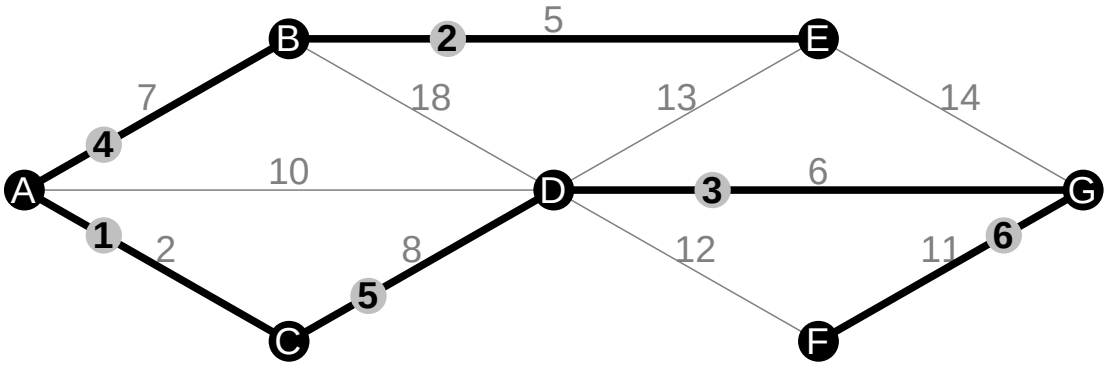
Arcs: AB, BD, DE, EC, EF.
Total length=23

3: a)



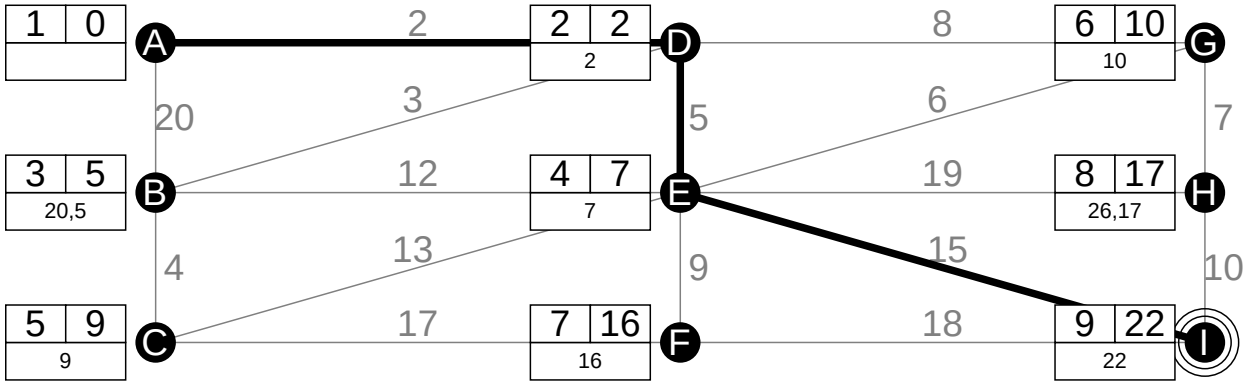
Arcs: AC, GJ, BD, AB, JH, GF, EG, BE, FI. Total length=34

b)



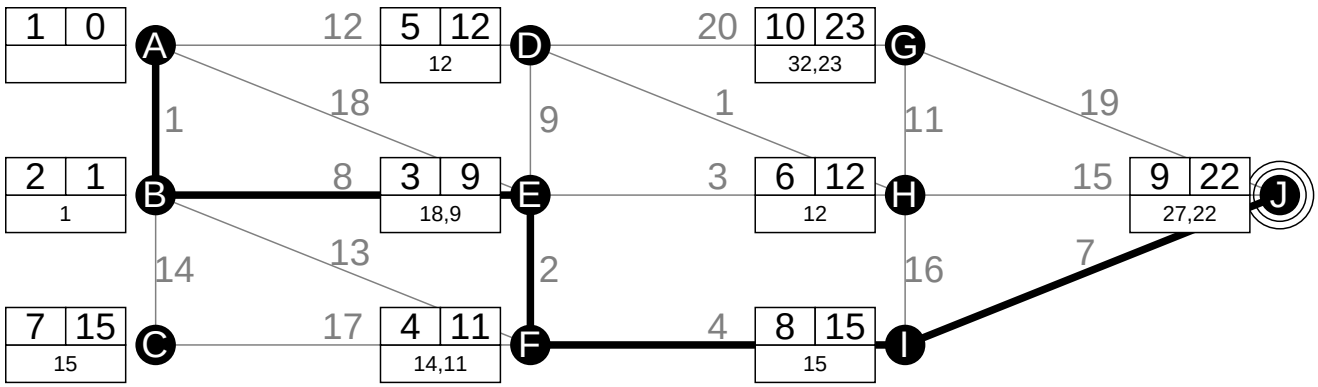
Arcs: AC, BE, DG, AB, CD, GF. Total length=39

4: a)



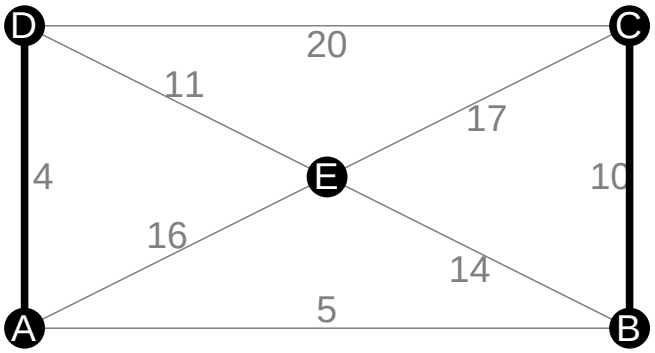
Route: ADEI. Length=22

b)



Route: ABEFIJ. Length=22

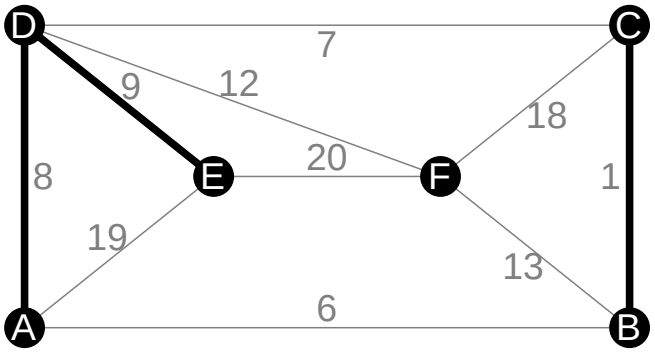
5: a)



Pairings: AB: 5 AC: 15 AD: 4
CD: 19 BD: 9 BC: 10
Tot: 24 Tot: 24 Tot: **14**

Poss. route: ABCBEADCEDA
Length: 97 + 14 = 111

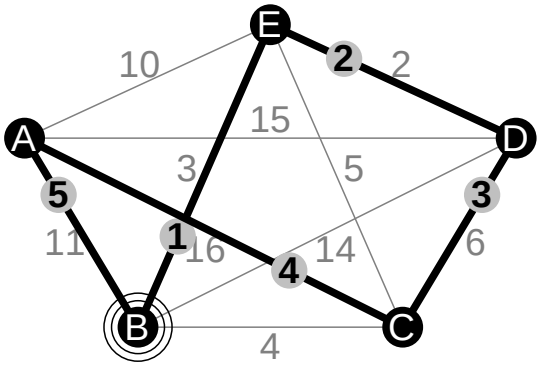
b)



Pairings: AB: 6 AC: 7 AE: 17
CE: 16 BE: 17 BC: 1
Tot: 22 Tot: 24 Tot: **18**

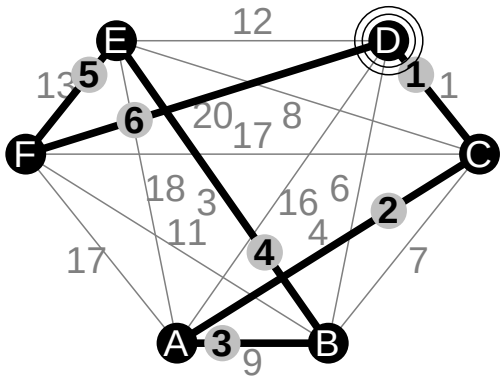
Poss. route: ABCBFCDADEDFEA
Length: 113 + 18 = 131

6: a)



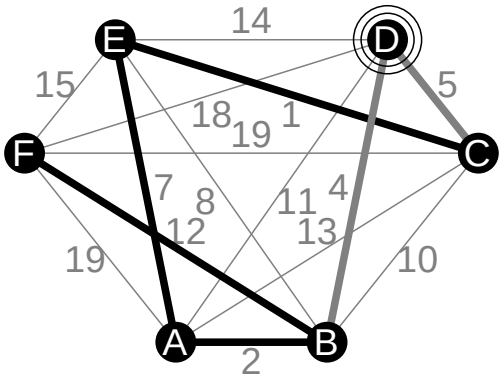
Cycle:BEDCAB. Length=38

b)



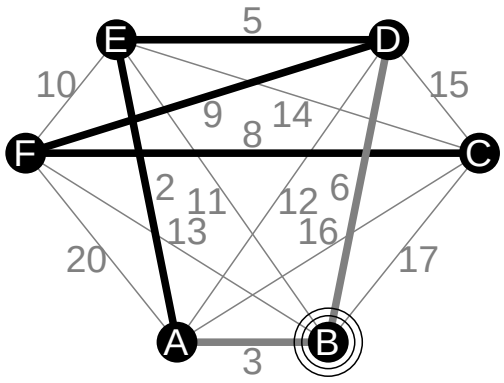
Cycle:DCABEFD. Length=50

7: a)



Length = 9 + 22 = 31

b)



Length = 9 + 24 = 33

8: a) 6 6 2 3 12 11 1 19
(Bin size 20)
Bin 1: 6 6 2 3 1 [18]
Bin 2: 12 [12]
Bin 3: 11 [11]
Bin 4: 19 [19]

b) 2 1 1 9 6 7 4 6
(Bin size 10)
Bin 1: 2 1 1 6 [10]
Bin 2: 9 [9]
Bin 3: 7 [7]
Bin 4: 4 6 [10]

9: a) 14 14 13 8 8 8 6 4 4 4 3 2 2
(Bin size 18)
Bin 1: 14 4 [18]
Bin 2: 14 4 [18]
Bin 3: 13 4 [17]
Bin 4: 8 8 2 [18]
Bin 5: 8 6 3 [17]
Bin 6: 2 [2]

b) 9 8 6 6 4 4 3 3 3 2
(Bin size 12)
Bin 1: 9 3 [12]
Bin 2: 8 4 [12]
Bin 3: 6 6 [12]
Bin 4: 4 3 3 2 [12]

10: a) 8 3 1 1 9 6 5

(Bin size 11)

Bin 1: 8 3 [11]

Bin 2: 1 1 9 [11]

Bin 3: 6 5 [11]

b) 13 1 8 6 5 2 7 4 1 9 8 6 3

(Bin size 14)

Bin 1: 13 1 [14]

Bin 2: 8 6 [14]

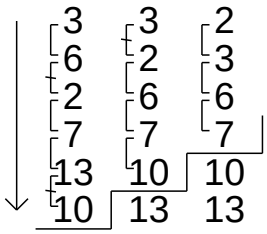
Bin 3: 5 2 7 [14]

Bin 4: 4 1 9 [14]

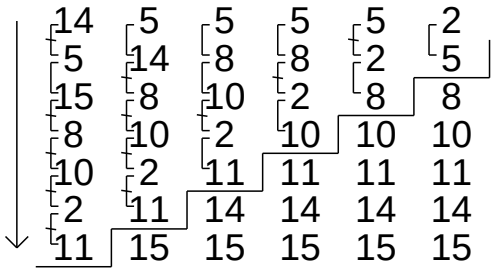
Bin 5: 8 6 [14]

Bin 6: 3 [3]

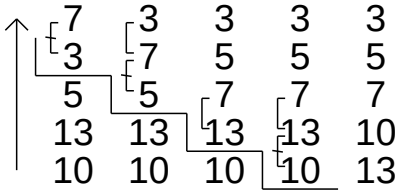
11: a)



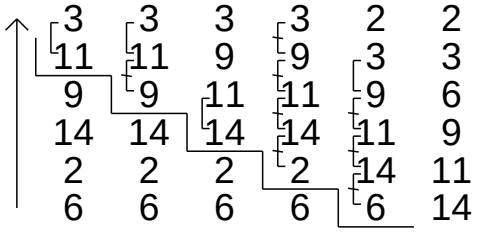
b)



12: a)



b)



13: a) Order $n!$

b) Order n^9

14: a) 40 seconds

b) 472392 seconds

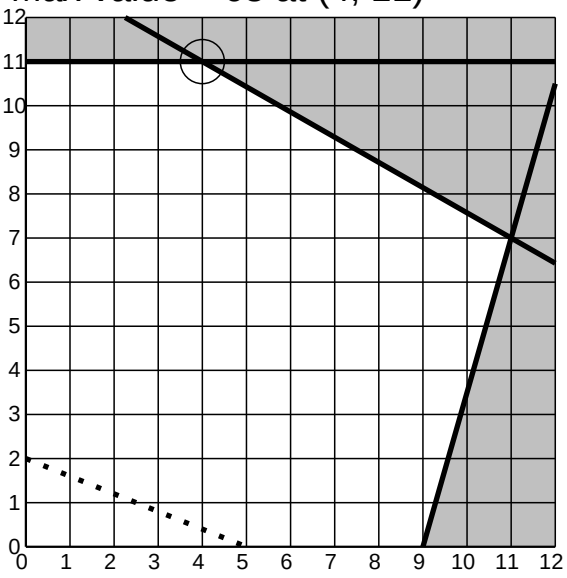
15: a) Minimise

b) Linear Programming (LP) formulation

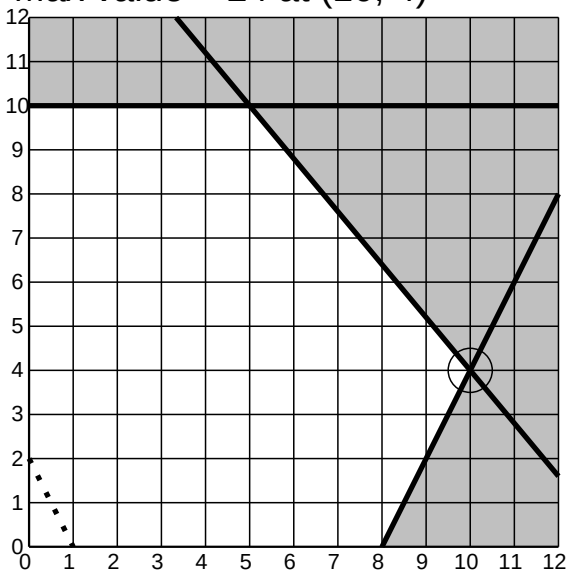
c) Objective function

d) Control variable

16: a) Max value = 63 at (4, 11)



b) Max value = 24 at (10, 4)



17: a) Iteration 1:

P	*x*	*y*	*s*	*t*	value
0	–4	3	1	0	6
0	6	3	0	1	36
1	–36	–18	0	0	32

Optimal solution:

P	*x*	*y*	*s*	*t*	value
0	0	5	1	$\frac{2}{3}$	30
0	1	$\frac{1}{2}$	0	$\frac{1}{6}$	6
1	0	0	0	6	248

$P = 248$, $x = 6$, $y = 0$

b) Iteration 1:

P	*x*	*y*	*s*	*t*	value
0	2	8	1	0	64
0	6	–7	0	1	6
1	–18	–40	0	0	27

Iteration 2:

P	*x*	*y*	*s*	*t*	value
0	$\frac{1}{4}$	1	$\frac{1}{8}$	0	8
0	$7\frac{3}{4}$	0	$\frac{7}{8}$	1	62
1	–8	0	5	0	347

Optimal solution:

P	*x*	*y*	*s*	*t*	value
0	0	1	$\frac{3}{31}$	$-\frac{1}{31}$	6
0	1	0	$\frac{7}{62}$	$\frac{4}{31}$	8
1	0	0	$5\frac{28}{31}$	$1\frac{1}{31}$	411

$P = 411$, $x = 8$, $y = 6$

18: a) Iteration 1:

P	x	y	z	s	t	u	value
0	4	30	15	1	0	0	6
0	4	20	12	0	1	0	27
0	8	4	36	0	0	1	36
1	–6	–14	–9	0	0	0	36

Iteration 2:

P	x	y	z	s	t	u	value
0	$\frac{2}{15}$	1	$\frac{1}{2}$	$\frac{1}{30}$	0	0	$\frac{1}{5}$
0	$1\frac{1}{3}$	0	2	$-\frac{2}{3}$	1	0	23
0	$7\frac{7}{15}$	0	34	$-\frac{2}{15}$	0	1	$35\frac{1}{5}$
1	$-4\frac{2}{15}$	0	–2	$\frac{7}{15}$	0	0	$38\frac{4}{5}$

Optimal solution:

P	x	y	z	s	t	u	value
0	1	$7\frac{1}{2}$	$3\frac{3}{4}$	$\frac{1}{4}$	0	0	$1\frac{1}{2}$
0	0	–10	–3	–1	1	0	21
0	0	–56	6	–2	0	1	24
1	0	31	$13\frac{1}{2}$	$1\frac{1}{2}$	0	0	45

$P = 45$, $x = 1\frac{1}{2}$, $y = 0$

b) Iteration 1:

P	x	y	z	s	t	u	value
0	10	24	10	1	0	0	40
0	16	(24)	40	0	1	0	2
0	9	10	9	0	0	1	21
1	–5	–15	–15	0	0	0	10

Optimal solution:

P	x	y	z	s	t	u	value
0	–6	0	–30	1	–1	0	38
0	$\frac{2}{3}$	1	$1\frac{2}{3}$	0	$\frac{1}{24}$	0	$\frac{1}{12}$
0	$2\frac{1}{3}$	0	$-7\frac{2}{3}$	0	$-\frac{5}{12}$	1	$20\frac{1}{6}$
1	5	0	10	0	$\frac{5}{8}$	0	$11\frac{1}{4}$

$P = 11\frac{1}{4}$, $x = 0$, $y = \frac{1}{12}$