Demo AS Discrete Questions

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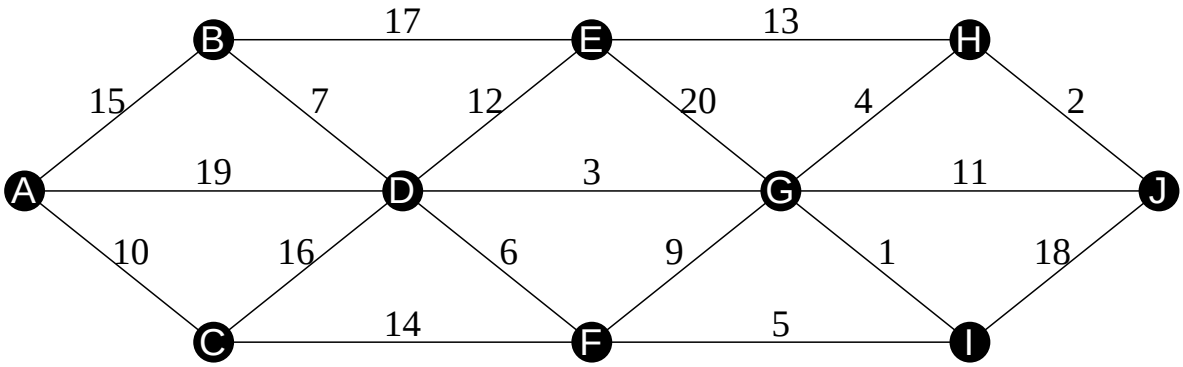
Please answer on file paper.

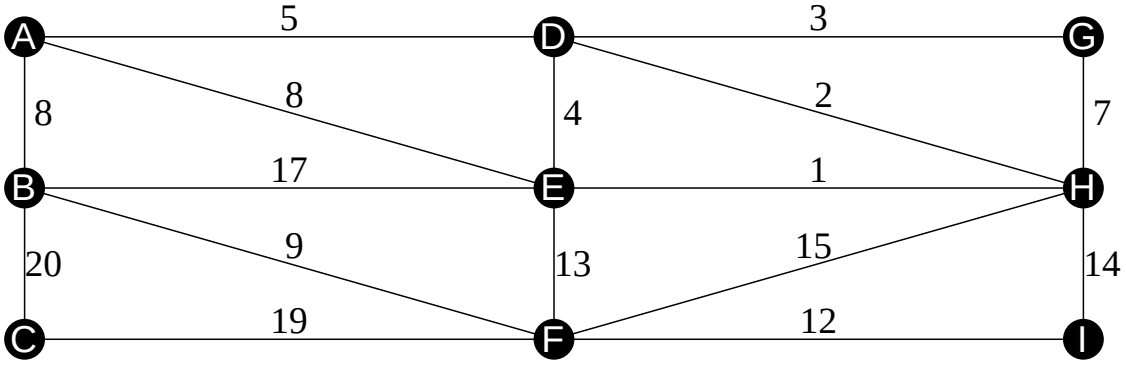
1: Find the Minimum Spanning Tree using Prim's Algorithm starting from vertex A:

a)



Arcs/Length:

b)



Arcs/Length:

2: Find the Minimum Spanning Tree using Prim's Algorithm starting from vertex A:

a)

	A	B	C	D	E	F
A	–	29	26	15	28	7
B	29	–	12	18	13	21
C	26	12	–	11	22	24
D	15	18	11	–	1	5
E	28	13	22	1	–	19
F	7	21	24	5	19	–

Arcs:

Total length=

b)

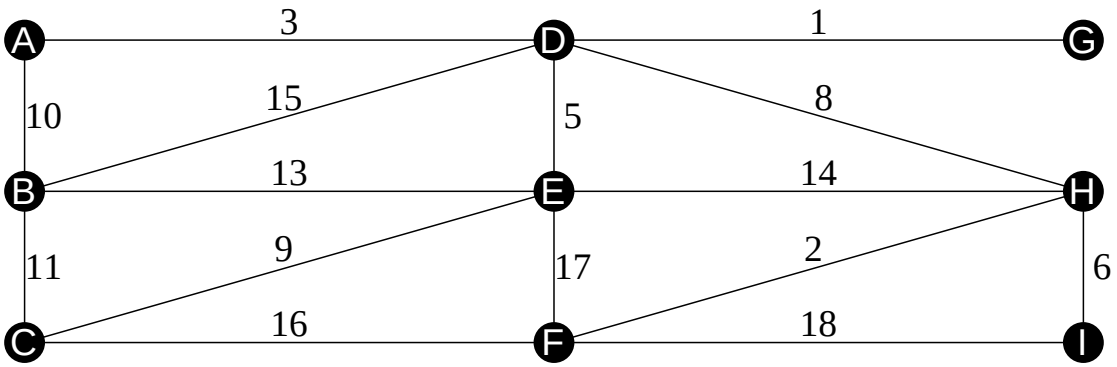
	A	B	C	D	E	F	G
A	–	2	17	14	8	25	30
B	2	–	4	3	6	20	23
C	17	4	–	27	9	16	10
D	14	3	27	–	19	18	21
E	8	6	9	19	–	24	12
F	25	20	16	18	24	–	29
G	30	23	10	21	12	29	–

Arcs:

Total length=

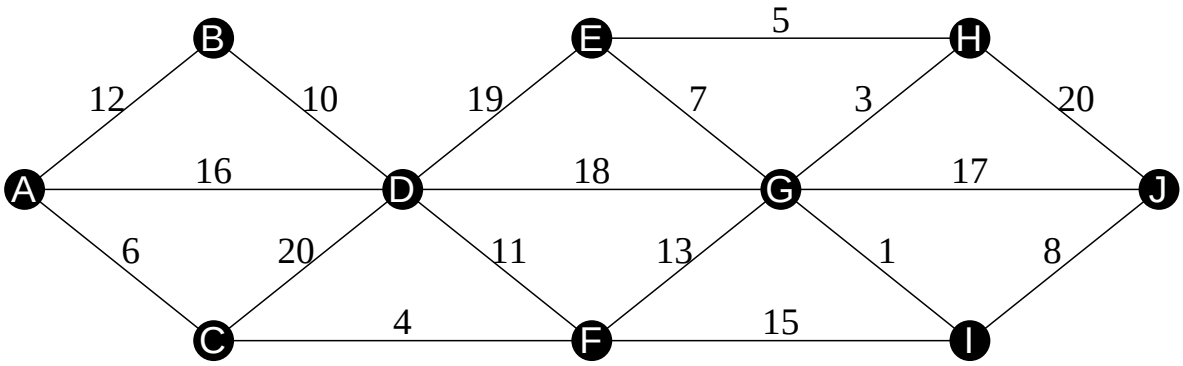
3: Find the Minimum Spanning Tree using Kruskal's Algorithm:

a)



Arcs/Length:

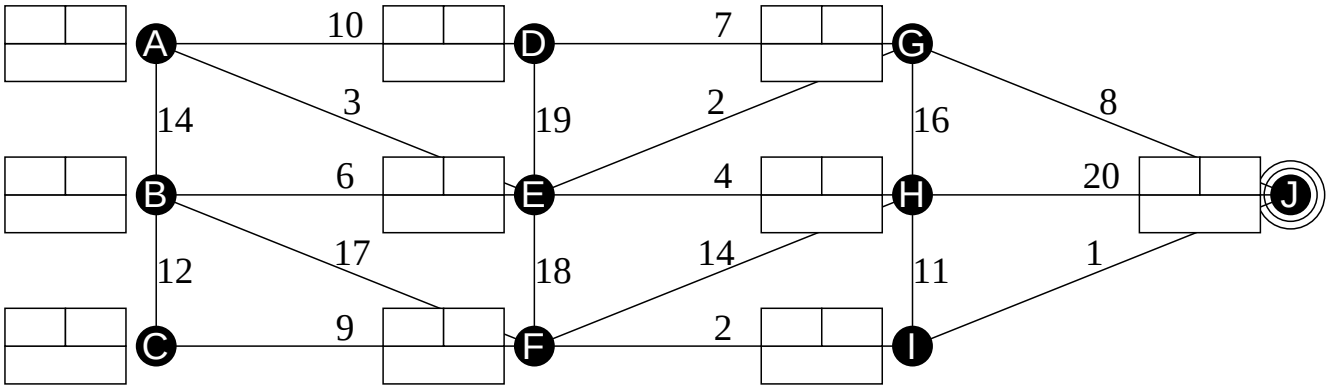
b)



Arcs/Length:

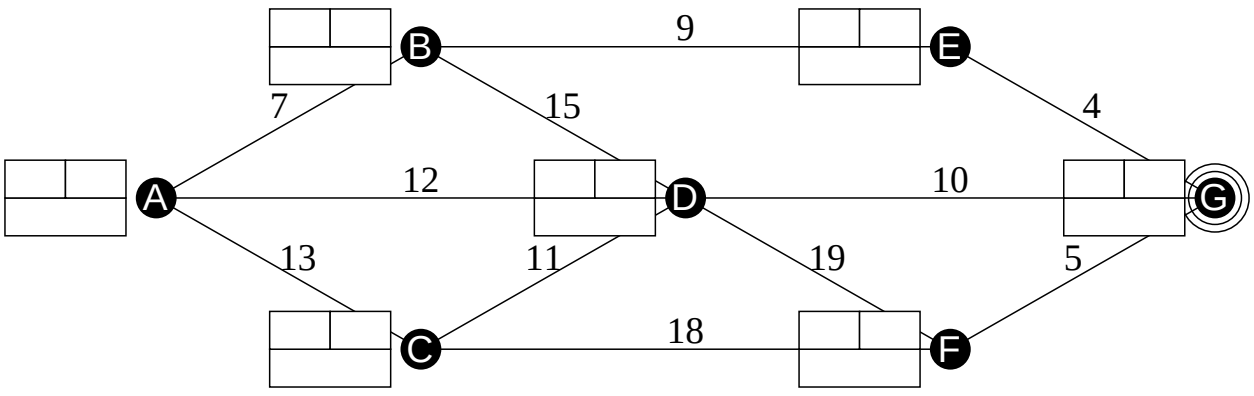
4: Find the shortest route from A to the ringed vertex using Dijkstra's Algorithm:

a)



Route/Length:

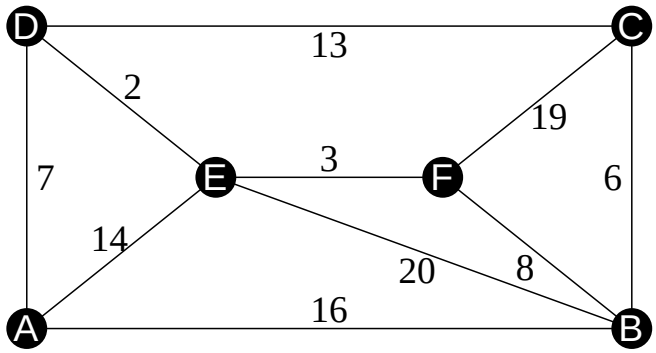
b)



Route/Length:

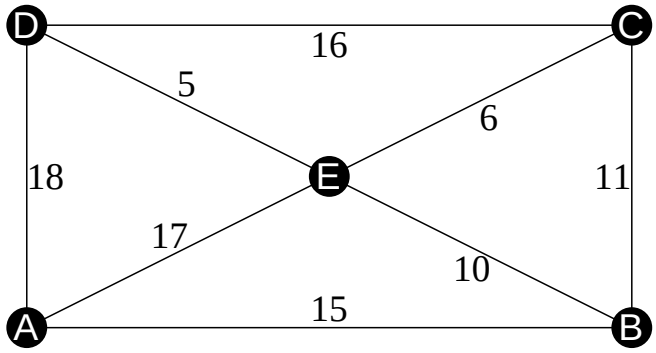
5: Find the shortest route starting and finishing at A which includes every edge.

a)



Pairings/Best solution:

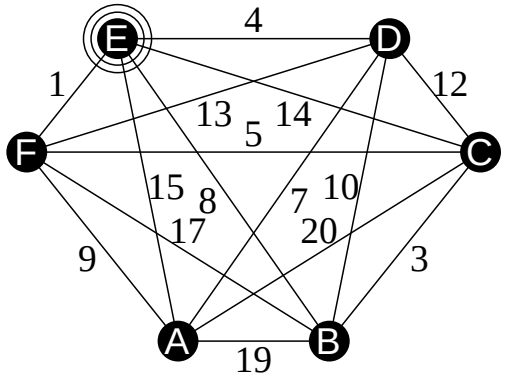
b)



Pairings/Best solution:

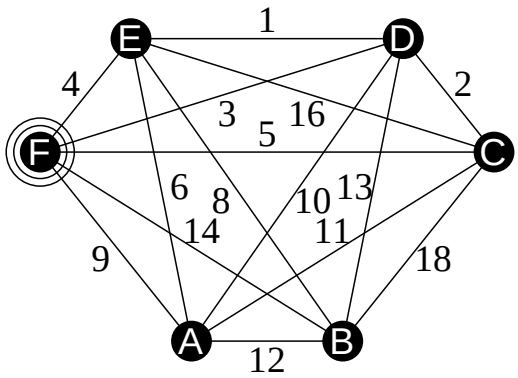
6: Find a Hamiltonian cycle using the Nearest Neighbour algorithm (start at the ringed vertex):

a)



Cycle/Length:

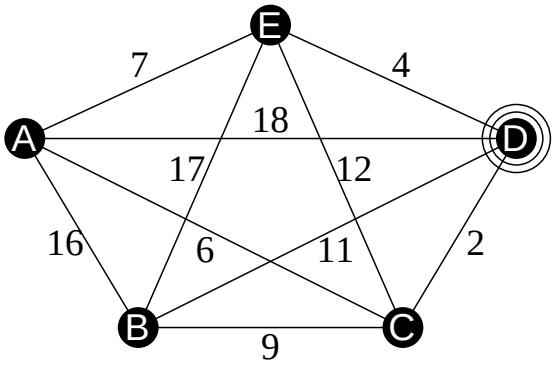
b)



Cycle/Length:

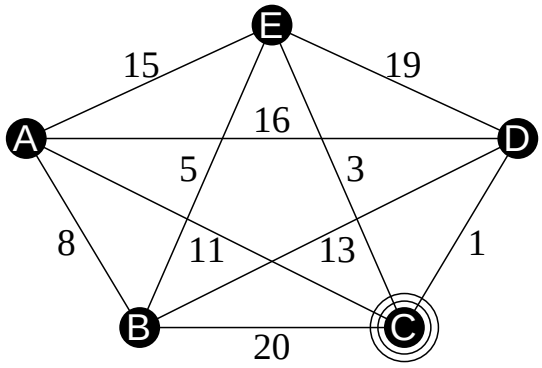
7: Find a lower bound for the Travelling Salesperson Problem (remove the ringed vertex):

a)



Lower Bound:

b)



Lower Bound:

8: Use the first fit algorithm to pack these objects into bins:

a) 3 17 12 15 4 1 2 3 4
(Bin size 19)

b) 10 5 4 9 2 8 4 12
(Bin size 18)

9: Use the first fit decreasing algorithm to pack these objects into bins:

a) 5 2 7 4 6 2 7 3 3 4 2 4 1 10
(Bin size 12)

b) 1 2 7 6 3 4 5 3 1 2 5 4
(Bin size 10)

10: Use the full bin algorithm to pack these objects into bins:

a) 9 11 1 19 13 7
(Bin size 20)

b) 9 6 11 4 1 4 10 2 7 13 5 9 2 5
(Bin size 15)

11: Arrange the following lists in ascending order using Bubble Sort:

a) 4 5 7 10 1 13

b) 10 13 14 4 9 6 1

12: Arrange the following lists in ascending order using Shuttle Sort:

a) 3 4 1 6 7

b) 9 5 8 2 1 3

13: Find the order of each algorithm, given its efficiency:

a) Time taken = $9n^{10}$

b) Time taken = $4^n + 14n + 17n^3 + 5n!$

14: Solve the following:

a) An algorithm has order n^4 and takes 486 seconds to solve a problem of size 3.
Estimate the time taken to solve a problem of size 7.

b) An algorithm has order 4^n and takes 144 seconds to solve a problem of size 2.
Estimate the time taken to solve a problem of size 10.

15: Give the following Linear Programming terms:

a) Find the largest value of

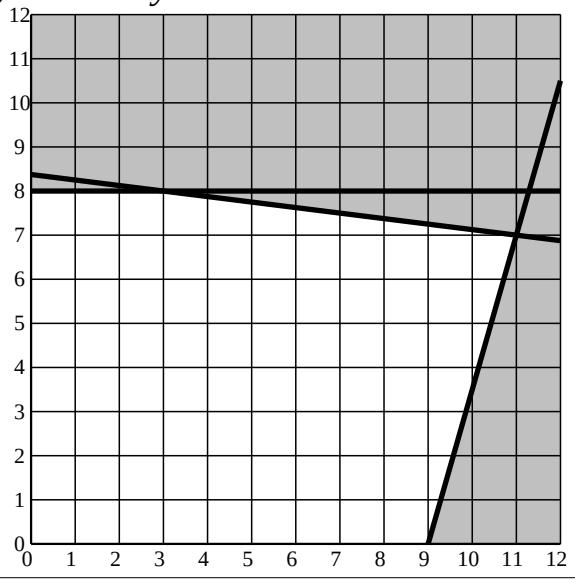
b) An inequality corresponding to a restriction on the problem

c) A corner of the region of allowable points

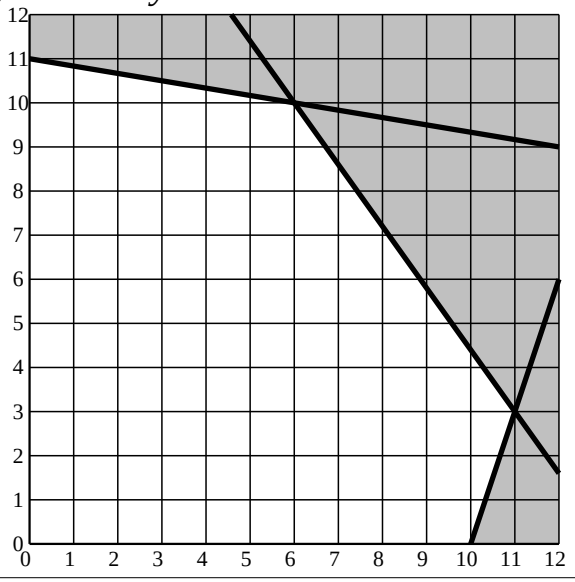
d) Find the smallest value of

16: Maximise each objective function:

a) $P = 4x + y$



b) $P = x + 3y$



17: Solve using the Simplex algorithm:

a)

P	x	y	s	t	value
0	0	8	1	0	8
0	1	–6	0	1	2
1	–6	–12	0	0	28

b)

P	x	y	s	t	value
0	–1	9	1	0	27
0	4	0	0	1	36
1	–15	–45	0	0	6

18: Solve using the Simplex algorithm:

a)

P	x	y	z	s	t	u	value
0	7	4	32	1	0	0	4
0	1	32	36	0	1	0	4
0	2	50	10	0	0	1	10
1	–30	–6	–18	0	0	0	12

b)

P	x	y	z	s	t	u	value
0	20	5	4	1	0	0	9
0	20	9	5	0	1	0	8
0	3	24	12	0	0	1	20
1	–10	–8	–10	0	0	0	20

Answers: Demo AS Discrete Questions

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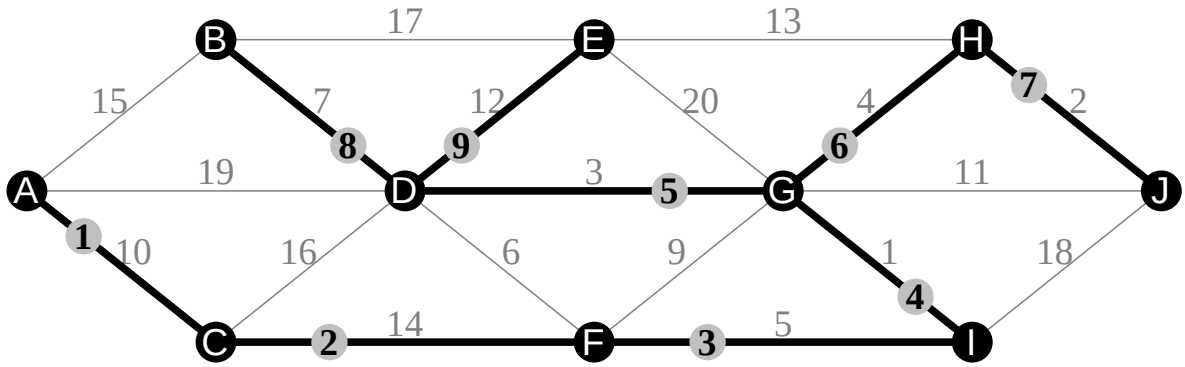
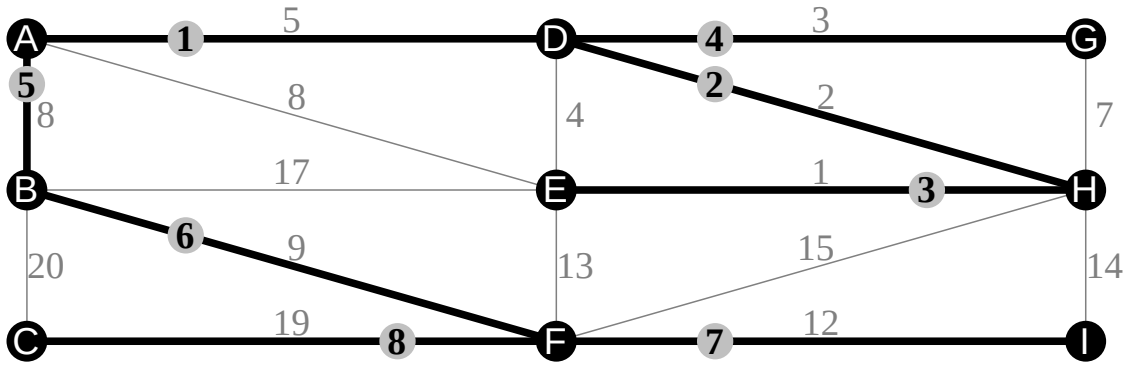
1: a)



Arcs: AC, CF, FI, IG, GD, GH, HJ, DB, DE. Total length=58

b)



Arcs: AD, DH, HE, DG, AB, BF, FI, FC. Total length=59

2: a)

	A (1)	B (6)	C (5)	D (3)	E (4)	F (2)
A	–	29	26	15	28	7
B	29	–	(12)	18	13	21
C	26	12	–	(11)	22	24
D	15	18	11	–	1	(5)
E	28	13	22	(1)	–	19
F	(7)	21	24	5	19	–

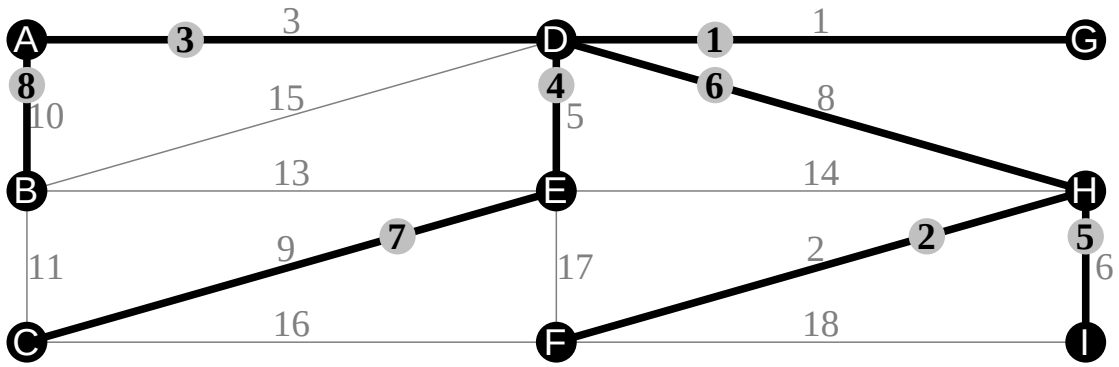
Arcs: AF, FD, DE, DC, CB.
Total length=36

b)

	A (1)	B (2)	C (4)	D (3)	E (5)	F (7)	G (6)
A	–	2	17	14	8	25	30
B	(2)	–	4	3	6	20	23
C	17	(4)	–	27	9	16	10
D	14	(3)	27	–	19	18	21
E	8	(6)	9	19	–	24	12
F	25	20	(16)	18	24	–	29
G	30	23	(10)	21	12	29	–

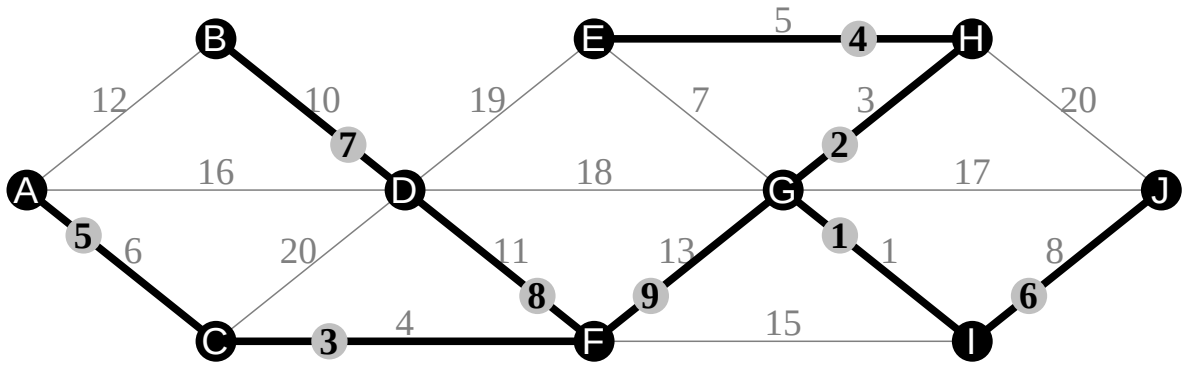
Arcs: AB, BD, BC, BE, CG, CF.
Total length=41

3: a)



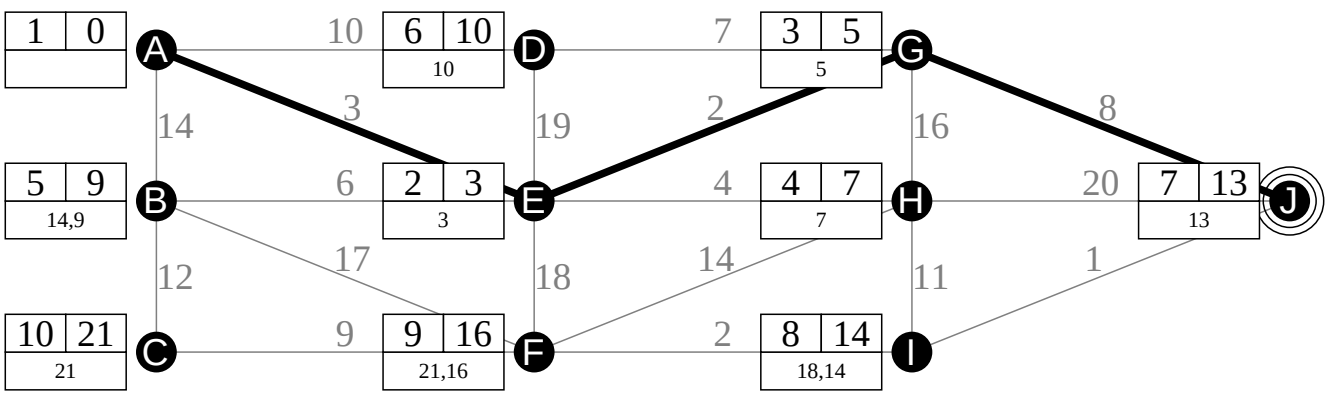
Arcs: DG, HF, AD, DE, HI, DH, EC, AB. Total length=44

b)



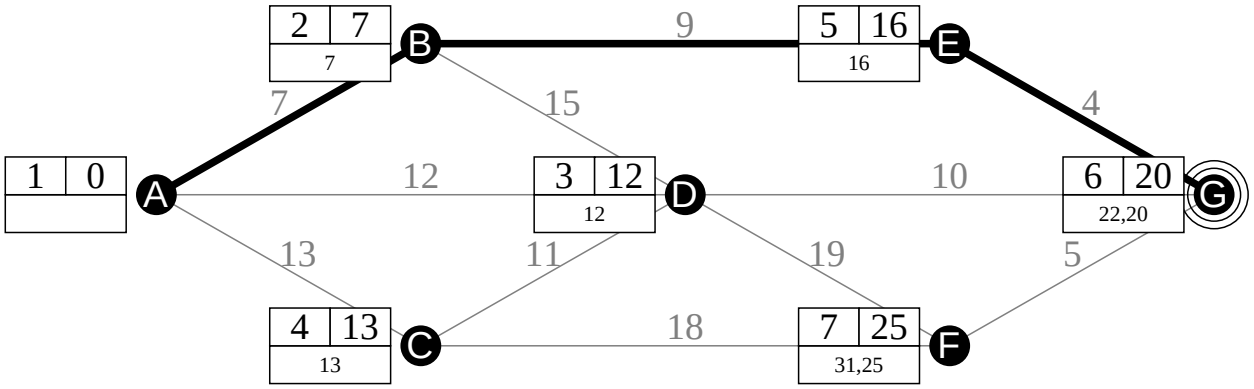
Arcs: GI, GH, CF, HE, AC, IJ, DB, FD, FG. Total length=61

4: a)



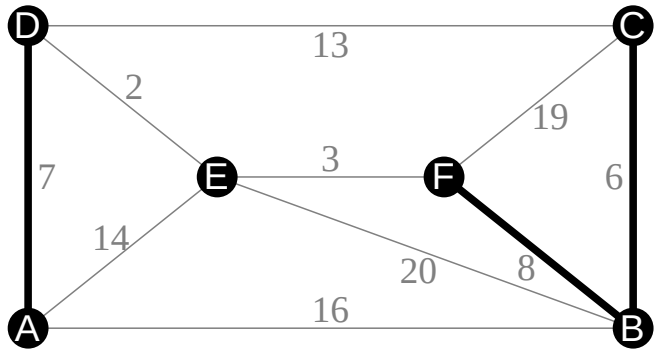
Route: AEGJ. Length=13

b)



Route: ABEG. Length=20

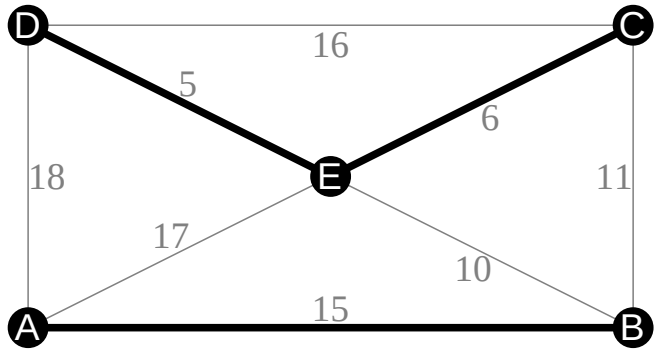
5: a)



Pairings: AC: 20 AD: 7 AF: 12
DF: 5 CF: 14 CD: 13
Tot: 25 Tot: **21** Tot: 25

Poss. route: ABCBEADCFBFEDA
Length: 108 + 21 = 129

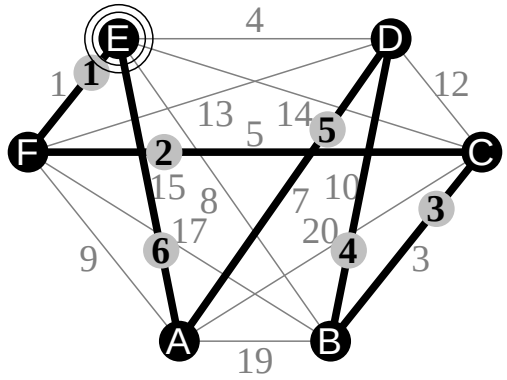
b)



Pairings: AB: 15 AC: 23 AD: 18
CD: 11 BD: 15 BC: 11
Tot: **26** Tot: 38 Tot: 29

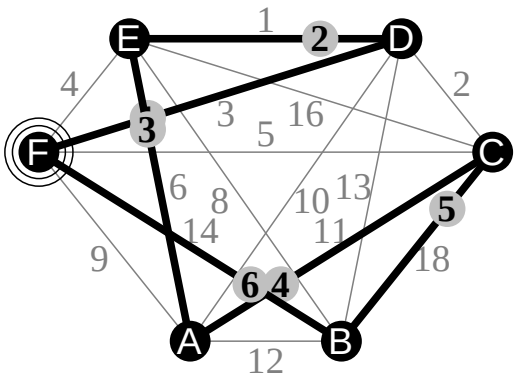
Poss. route: ABADCBECEDEA
Length: 98 + 26 = 124

6: a)



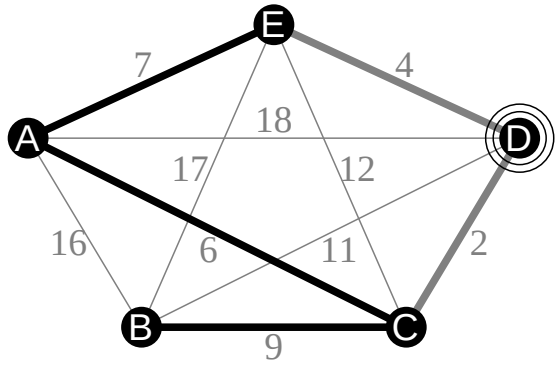
Cycle:EFCBDAE. Length=41

b)



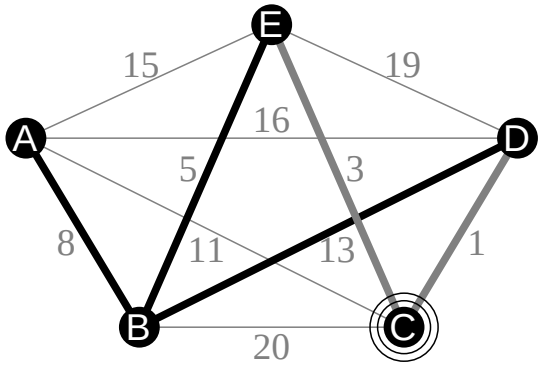
Cycle:FDEACBF. Length=53

7: a)



Length = 6 + 22 = 28

b)



Length = 4 + 26 = 30

8: a) 3 17 12 15 4 1 2 3 4
(Bin size 19)
Bin 1: 3 12 4 [19]
Bin 2: 17 1 [18]
Bin 3: 15 2 [17]
Bin 4: 3 4 [7]

b) 10 5 4 9 2 8 4 12
(Bin size 18)
Bin 1: 10 5 2 [17]
Bin 2: 4 9 4 [17]
Bin 3: 8 [8]
Bin 4: 12 [12]

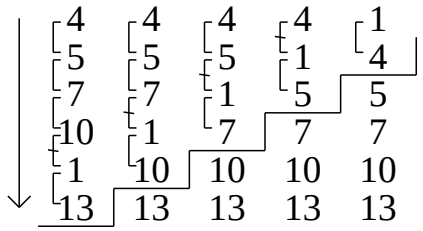
9: a) 10 7 7 6 5 4 4 4 3 3 2 2 2 1
(Bin size 12)
Bin 1: 10 2 [12]
Bin 2: 7 5 [12]
Bin 3: 7 4 1 [12]
Bin 4: 6 4 2 [12]
Bin 5: 4 3 3 2 [12]

b) 7 6 5 5 4 4 3 3 2 2 1 1
(Bin size 10)
Bin 1: 7 3 [10]
Bin 2: 6 4 [10]
Bin 3: 5 5 [10]
Bin 4: 4 3 2 1 [10]
Bin 5: 2 1 [3]

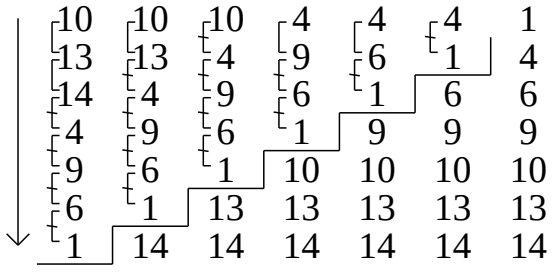
10: a) 13 7 11 9 19 1
(Bin size 20)
Bin 1: 13 7 [20]
Bin 2: 11 9 [20]
Bin 3: 19 1 [20]

b) 5 1 9 2 9 4 6 7 2 10 5 11 4 13
(Bin size 15)
Bin 1: 5 1 9 [15]
Bin 2: 2 9 4 [15]
Bin 3: 6 7 2 [15]
Bin 4: 10 5 [15]
Bin 5: 11 4 [15]
Bin 6: 13 [13]

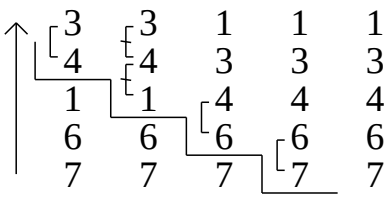
11: a)



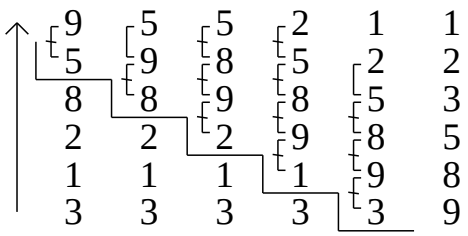
b)



12: a)



b)

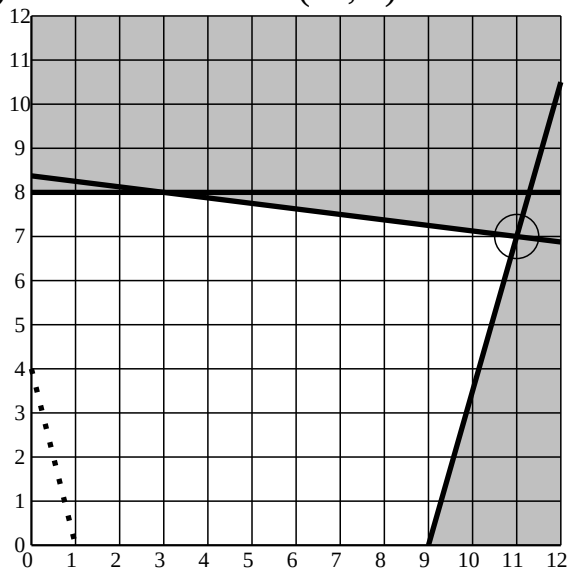


13: a) Order n^{10} b) Order $n!$

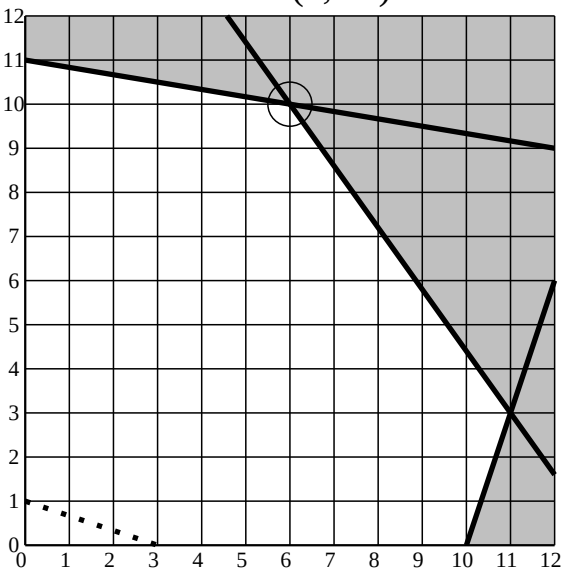
14: a) 14406 seconds b) 9437184 seconds

15: a) Maximise b) Constraint c) Vertex d) Minimise

16: a) Max value = 51 at (11, 7)



b) Max value = 36 at (6, 10)



17: a) Iteration 1:

P	x	y	s	t	value
0	0	(8)	1	0	8
0	1	−6	0	1	2
1	−6	−12	0	0	28

Iteration 2:

P	x	y	s	t	value
0	0	1	$\frac{1}{8}$	0	1
0	(1)	0	$\frac{3}{4}$	1	8
1	−6	0	$1\frac{1}{2}$	0	40

Optimal solution:

P	x	y	s	t	value
0	0	1	$\frac{1}{8}$	0	1
0	1	0	$\frac{3}{4}$	1	8
1	0	0	6	6	88

$P = 88$, $x = 8$, $y = 1$

b) Iteration 1:

P	x	y	s	t	value
0	−1	(9)	1	0	27
0	4	0	0	1	36
1	−15	−45	0	0	6

Iteration 2:

P	x	y	s	t	value
0	$-\frac{1}{9}$	1	$\frac{1}{9}$	0	3
0	(4)	0	0	1	36
1	−20	0	5	0	141

Optimal solution:

P	x	y	s	t	value
0	0	1	$\frac{1}{9}$	$\frac{1}{36}$	4
0	1	0	0	$\frac{1}{4}$	9
1	0	0	5	5	321

$P = 321$, $x = 9$, $y = 4$

18: a) Iteration 1:

P	x	y	z	s	t	u	value
0	(7)	4	32	1	0	0	4
0	1	32	36	0	1	0	4
0	2	50	10	0	0	1	10
1	−30	−6	−18	0	0	0	12

Optimal solution:

P	x	y	z	s	t	u	value
0	1	$\frac{4}{7}$	$4\frac{4}{7}$	$\frac{1}{7}$	0	0	$\frac{4}{7}$
0	0	$31\frac{3}{7}$	$31\frac{3}{7}$	$-\frac{1}{7}$	1	0	$3\frac{3}{7}$
0	0	$48\frac{6}{7}$	$\frac{6}{7}$	$-\frac{2}{7}$	0	1	$8\frac{6}{7}$
1	0	$11\frac{1}{7}$	$119\frac{1}{7}$	$4\frac{2}{7}$	0	0	$29\frac{1}{7}$

$P = 29\frac{1}{7}$, $x = \frac{4}{7}$, $y = 0$

b) Iteration 1:

P	x	y	z	s	t	u	value
0	20	5	4	1	0	0	9
0	(20)	9	5	0	1	0	8
0	3	24	12	0	0	1	20
1	−10	−8	−10	0	0	0	20

Iteration 2:

P	x	y	z	s	t	u	value
0	0	−4	−1	1	−1	0	1
0	1	$\frac{9}{20}$	($\frac{1}{4}$)	0	$\frac{1}{20}$	0	$\frac{2}{5}$
0	0	$22\frac{13}{20}$	$11\frac{1}{4}$	0	$-\frac{3}{20}$	1	$18\frac{4}{5}$
1	0	$-3\frac{1}{2}$	$-7\frac{1}{2}$	0	$\frac{1}{2}$	0	24

Optimal solution:

P	x	y	z	s	t	u	value
0	4	$-2\frac{1}{5}$	0	1	$-\frac{4}{5}$	0	$2\frac{3}{5}$
0	4	$1\frac{4}{5}$	1	0	$\frac{1}{5}$	0	$1\frac{3}{5}$
0	−45	$2\frac{2}{5}$	0	0	$-2\frac{2}{5}$	1	$\frac{4}{5}$
1	30	10	0	0	2	0	36

$P = 36$, $x = 0$, $y = 0$